Performance Comparisons of Optimal Routing by Pipe, Hose, and Intermediate Models

Eiji Oki and Ayako Iwaki
Dept. of Information and Communication Engineering
The University of Electro-Communications
Outline

- Background on optimal IP routing
- Conventional traffic models
- Introduction of intermediate model
- Optimal routing formulations
- Performance comparisons
- Conclusions
Optimal routing in IP networks

- What is good routing?
  - To utilize the network resources efficiently and increase the network throughput.

- Approaches
  - Explicit routing
    - Traffic demands are explicitly routed.
  - Load balancing
    - Traffic demands are split among source and destination nodes.

- Minimizing the network congestion ratio leads to increase additional admissible traffic.
  - Network congestion ratio
    - Maximum link load of all network links
  - This is a main target in this study.

Congestion
Optimal routing for traffic models

- Two popular models
  - Pipe model
  - Hose model
- Newly introduced
  - Intermediate model
Pipe model

- Assumption on traffic model
  - Traffic matrix, $T = \{d_{pq}\}$, is exactly known.
    - $d_{pq}$: traffic demand from source node $p$ to destination node $q$.

- Routing scheme
  - Traffic demands are assumed to be explicitly routed and flexibly split among source and destination nodes, by using Multi-Protocol Label Switching (MPLS) technologies

\[
T = \begin{pmatrix}
    d_{11} & d_{12} & d_{13} \\
    d_{21} & d_{22} & d_{23} \\
    d_{31} & d_{32} & d_{33}
\end{pmatrix}
\]
Pipe model (cont’d)

- Under the pipe model, the most efficient routing is achieved.
- However,
  - It is difficult for network operators to get an exact traffic matrix
  - Because
    - Measurement of traffic demand for each source and destination pair is difficult when the network size is large.
    - Traffic demand is often fluctuated.

Measurement of traffic demand

\[
T = \begin{pmatrix}
  d_{11} & d_{12} & d_{13} \\
  d_{21} & d_{22} & d_{23} \\
  d_{31} & d_{32} & d_{33}
\end{pmatrix}
\]

Fluctuation of traffic demand

![Graph showing fluctuation of traffic demand over time]
Hose model

- Easy for network operators to specify the traffic as only the total outgoing/incoming traffic from/to edge node $p$ and edge node $q$.
- The hose model is specified by:

\[
\begin{align*}
\text{Incoming traffic} & \quad \sum_q d_{pq} \leq \alpha_p \\
\text{Outgoing traffic} & \quad \sum_p d_{pq} \leq \beta_q
\end{align*}
\]
Hose model (cont’d)

- Its routing performance is much lower than that of the pipe model.
  - All possible set of traffic matrix specified by the hose model must be considered.
- To enhance the routing performance
  - It is desirable for network operators to narrow the range of traffic conditions specified by hose model.
Intermediate model introduced

- Network operators impose additional bounds on the hose model from their operational experience and past traffic data.
- **Add lower and upper bounds** to the hose model.

\[
\sum_{q} d_{pq} \leq \alpha_p \\
\sum_{p} d_{pq} \leq \beta_q
\]

Lower bound $\delta_{pq} \leq d_{pq} \leq \gamma_{pq}$

Routing performance
- High: Pipe model
- Low: Hose model

Range of traffic matrix
- Narrow
- Wide
Example for range of $d_{pq}$

- A range of $d_{pq}$ is obtained from network operators’ operational experience and past traffic data.
Pipe, hose, and intermediate models

Questions

- How much differences of optimal-routing performances are there between the pipe and hose models?
- How much does the intermediate model improve the performance compared to the hose model?
Optimal routing problems

- **Pipe model**
  - Wang et al. formulated an optimal routing problem as a linear programming (LP) formulation [ICCCN 99].
  - Was solved.

- **Hose model**
  - J. Chu et al. formulated an optimal routing problem as an LP formulation [ICC 07].
  - Was solved.

- **Intermediate model**
  - An optimization problem extended from the pipe model is not able to be solved as a regular LP problem.
  - We show how to solve it.
Network model

\( G(V, E) \) : directed graph to represent the network

\( V \) : set of vertexes, \( Q \subseteq V \) : set of edge nodes

\( E \) : set of links,

\( x_{ij}^{pq} \) : portion of traffic from node \( p \in Q \) to node \( q \in Q \) through link \( (i, j) \in E \)

\( c_{ij} \) : capacity of link \( (i, j) \in E \)

\( T = \{ d_{pq} \} \) : traffic demand

\( r \) : network cогestion ratio,

maximum value of all link utilization rates in the network
Optimal routing with pipe model

Optimization problem

Objective: \( \min r \) \hspace{1cm} (1a)

Constraints:

\[
\begin{align*}
\sum_{j:(i,j) \in E} x_{ij}^{pq} - \sum_{j:(j,i) \in E} x_{ji}^{pq} &= 0, \ p, q \in Q, i \neq p, i \neq q \hspace{1cm} (1b) \\
\sum_{j:(i,j) \in E} x_{ij}^{pq} - \sum_{j:(j,i) \in E} x_{ji}^{pq} &= 1, \ p, q \in Q, i = p \hspace{1cm} (1c) \\
\sum_{p,q \in Q} d_{pq} x_{ij}^{pq} &\leq c_{ij} r, (i, j) \in E \hspace{1cm} (1d) \\
0 &\leq x_{ij}^{pq} \leq 1, \ p, q \in Q, (i, j) \in E \hspace{1cm} (1e) \\
0 &\leq r \leq 1 \hspace{1cm} (1f)
\end{align*}
\]

Given parameters: \( d_{pq}, c_{ij} \)

Decision variables: \( r, x_{ij}^{pq} \)

This is a linear programming (LP) problem.
Optimal routing with intermediate model

### Optimization problem

**Objective:** \( \text{min } r \)  

**Constraints:**
\[
\begin{align*}
\sum_{j:(i,j) \in E} x_{ij}^{pq} - \sum_{j:(j,i) \in E} x_{ji}^{pq} &= 0, \ p, q \in Q, i \neq p, i \neq q \quad (1a) \\
\sum_{j:(i,j) \in E} x_{ij}^{pq} - \sum_{i:(i,i) \in E} x_{ji}^{pq} &= 1, \ p, q \in Q, i = p \quad (1b) \\
\sum_{p,q} d_{pq} x_{ij}^{pq} &\leq c_{ij} r, \ (i, j) \in E \quad (1c) \\
0 &\leq x_{ij}^{pq} \leq 1, \ p, q \in Q, (i, j) \in E \quad (1d) \\
0 &\leq r \leq 1 \quad (1e)
\end{align*}
\]

**Given parameters:** \( d_{pq}, c_{ij} \)

**Decision variables:** \( r, x_{ij}^{pq} \)

The range of \( d_{pq} \) is given by:
\[
\begin{align*}
\sum_{q} d_{pq} &\leq \alpha_p \quad (1g) \\
\sum_{p} d_{pq} &\leq \beta_q \quad (1h) \\
\delta_{pq} &\leq d_{pq} \leq \gamma_{pq} \quad (1i)
\end{align*}
\]

This optimization problem is a linear programming (LP) one. However, it is impossible to consider all possible combinations of \( d_{pq} \) specified by Eqs. (1g)-(1i).
Optimal routing with intermediate model (cont’d)

- The optimization problem is solved by the following property, which is obtained by introducing the dual theorem and extending Chu’s property in the hose model to the intermediate model [ICC 07].

- Property: $x_{ij}^{pq}$ achieves congestion ratio $\leq r$ for all traffic matrices constrained by the intermediate model if and only if there exist the following non-negative parameters such that

\[
\begin{align*}
\text{i)} \quad & \sum_{p \in Q} \alpha_p \pi_{ij}(p) + \sum_{p \in Q} \beta_p \lambda_{ij}(p) + \sum_{p, q \in Q} [\gamma_{pq} \eta_{ij}(p, q) - \delta_{pq} \theta_{ij}(p, q)] \leq c_{ij} r, \\
& \quad \text{for each } (i, j) \in E \\
\text{ii)} \quad & x_{ij}^{pq} \leq \pi_{ij}(p) + \lambda_{ij}(q) + \eta_{ij}(p, q) - \theta_{ij}(p, q), \\
& \quad \text{for each } (i, j) \in E \text{ and every } p, q \in Q
\end{align*}
\]
The optimal routing problem is transformed into the following regular LP formulation.

**Optimization problem**

**Objective**: \( \text{min} \ r \)

**Constraints**: 

\[
\sum_{j: (i,j) \in E} x_{ij}^{pq} - \sum_{j: (j,i) \in E} x_{ji}^{pq} = 0, \quad p, q \in Q, \quad i \neq p, i \neq q
\]

\[
\sum_{j: (i,j) \in E} x_{ij}^{pq} - \sum_{j: (j,i) \in E} x_{ji}^{pq} = 1, \quad p, q \in Q, \quad i = p
\]

\[
\sum_{p \in Q} \alpha_p \pi_{ij}(p) + \sum_{p \in Q} \beta_p \lambda_{ij}(p) + \sum_{p,q \in Q} [\gamma_{pq} \eta_{ij}(p,q) - \delta_{pq} \theta_{ij}(p,q)] \leq c_{ij}, \quad (i, j) \in E
\]

\[
x_{ij}^{pq} \leq \pi_{ij}(p) + \lambda_{ij}(q) + \eta_{ij}(p,q) - \theta_{ij}(p,q), \quad (i, j) \in E
\]

\[
\pi_{ij}(p), \lambda_{ij}(q), \eta_{ij}(p,q), \theta_{ij}(p,q) \geq 0
\]

\[
0 \leq x_{ij}^{pq} \leq 1, \quad p, q \in Q, \quad (i, j) \in E
\]

\[
0 \leq r \leq 1
\]
Performance evaluation

- Network congestion ratios are compared
  - Pipe, hose, and intermediate models

Simulation assumptions

- Link capacities randomly generated with uniform distribution in the range of (80,120)
- \(d_{pq}\) is randomly generated with uniform distribution in the range of (0,100)

\[
\alpha_p = \sum_q d_{pq}, \beta_q = \sum_p d_{pq}
\]

\[
\gamma_{pq} = \frac{1}{\mu} d_{pq}, \delta_{pq} = \nu d_{pq}
\]

\[
0 \leq \mu \leq 1, 0 \leq \nu \leq 1
\]

\((\mu, \nu) \to (1,1) : \text{pipe model}\)
\((\mu, \nu) \to (0,0) : \text{hose model}\)
Network models

- Sample networks

(a) Network 1  (b) Network 2  (c) Network 3
(d) Network 4  (e) Network 5

- Random networks
  - Randomly generated under the condition that average node degree $D$ is satisfied for a given number of nodes $N$. 
Comparisons of congestion ratio between pipe and hose models for sample networks.

The network congestion ratio for the pipe model is 30-40% lower than that of the hose model.
Comparisons of congestion ratio between pipe and hose models for random networks

- The difference of network congestion ratios is 27% to 45%.
Congestion ratios of intermediate model

The intermediate model reduces the network congestion ratio compared with the hose model, by narrowing the range of $d_{pq}$.

![Graph showing the comparison between Hose and Intermediate models]

- **Hose ($\mu = 0.0$, $\nu = 0.0$)**
- **Intermediate**
- **Pipe ($\mu = 1.0$, $\nu = 1.0$)

In the graph, the normalized congestion ratio is depicted against an X-axis representing $d_{pq}$. The range of $d_{pq}$ is shown to be narrowed in the intermediate model compared to the hose model.

Mathematically, it can be expressed as:

$$vd_{pq} \leq d_{pq} \leq \frac{1}{\mu}d_{pq}$$

with $0 \leq \mu \leq 1$ and $0 \leq \nu \leq 1$. 

The graph indicates the region where the normalized congestion ratio is between $0.6$ and $1.1$, showing a significant reduction in congestion.
Network operators’ choices on how to construct traffic models

- Choices considering between routing performance and operational cost
  - Case 1: routing performance is the most important factor.
    - they should narrow the range of traffic conditions as much as possible at the cost of their operational cost on traffic measurement and estimation.
  - Case 2: operational cost is the most important factor
    - The should not pay for the operational cost at the cost of routing performance.

- By introducing the intermediate model, network operators can specify the best traffic conditions
Conclusions

- The intermediate model was introduced.
  - The optimal routing problem was transformed into a regular LP problem.
- Comparisons of pipe, hose, and intermediate model
- Intermediate model
  - Lightens the difficulty of the pipe model
  - Narrows the range of traffic conditions from the hose model
  - Enhances the routing performance compared with the hose model.